A Tensor-Based Probabilistic Event Calculus

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Motivation for Tensor-Based Probabilistic Reasoning

- Robust and scalable logical inference in the presence of uncertainty.
- High rate and volume of input data.
- Exploit linear algebraic computation.
- Event Calculus allows expressive temporal reasoning.
- Perform CER by employing temporal windows and caching.

Event Calculus (Example)

$$
\begin{aligned}
&\mathsf{initiatedAt}(moving(P_1, P_2)\!=\!\mathsf{true},\ T) \leftarrow \\
&\quad \mathsf{happensAt}(walking(P_1),\ T), \\
&\quad \mathsf{happensAt}(walking(P_2),\ T), \\
&\quad \mathsf{holdsAt}(close(P_1, P_2)\!=\!\mathsf{true},\ T), \\
&\quad \mathsf{holdsAt}(similarOrientation(P_1, P_2)\!=\!\mathsf{true},\ T). \\
&\mathsf{terminatedAt}(moving(P_1, P_2)\!=\!\mathsf{true},\ T) \leftarrow \\
&\quad \mathsf{happensAt}(walking(P_1),\ T), \\
&\quad \mathsf{holdsAt}(close(P_1, P_2)\!=\!\mathsf{false},\ T).
\end{aligned}
$$

Event Calculus (*Law of inertia*)

$$\begin{array}{ll}
\mathsf{holdsAt}(\mathit{fl}(X,Y){=}v,\ T) \leftarrow & \mathsf{holdsAt}(\mathit{fl}(X,Y){=}v,\ T) \leftarrow \\
\quad \mathit{next}(T_{prev},\ T), & \quad \mathit{next}(T_{prev},\ T), \\
\quad \mathsf{initiatedAt}(\mathit{fl}(X,Y){=}v,\ T_{prev}). & \quad \mathsf{holdsAt}(\mathit{fl}(X,Y){=}v,\ T_{prev}), \\
 & \quad \mathsf{not}\ \ \mathsf{broken}(\mathit{fl}(X,Y){=}v,\ T_{prev}).
\end{array}$$

$$\begin{array}{ll}
\mathsf{broken}(\mathit{fl}(X,Y){=}v,\ T_{prev}) \leftarrow & \mathsf{broken}(\mathit{fl}(X,Y){=}v,\ T_{prev}) \leftarrow \\
\quad \mathsf{terminatedAt}(\mathit{fl}(X,Y){=}v,\ T_{prev}). & \quad \mathsf{initiatedAt}(\mathit{fl}(X,Y){=}v',\ T_{prev}),\ \ v \neq v'.
\end{array}$$

Probabilistic Event Calculus

- *Possible world* semantics.
- A probabilistic fact is denoted by $p :: f$, $p \in [0, 1]$.
- If f is not included in a world, then $\neg f$ is included with probability $1 - p$.
- The product of the probabilities of the facts (positive or negative) is the probability of the world.

Probabilistic Event Calculus (d-DNNF)

$$
\begin{array}{ll}
h^{t+1} & \rightarrow \textsf{holdsAt}(\mathit{moving}(c_1, c_2)\!=\!\textsf{true},\ t+1) \\
0.2 :: h^{t} & \rightarrow \textsf{holdsAt}(\mathit{moving}(c_1, c_2)\!=\!\textsf{true},\ t) \\
1.0 :: u^{t+1} & \rightarrow \mathit{next}(t, t+1) \\
0.3 :: wlk_1^{t} & \rightarrow \textsf{happensAt}(\mathit{walking}(c_1),\ t) \\
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1.0 :: cls_T^{t} & \rightarrow \textsf{holdsAt}(\mathit{close}(c_1, c_2)\!=\!\textsf{true},\ t) \\
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1.0 :: so^{t} & \rightarrow \textsf{holdsAt}(\mathit{similarOrientation}(c_1, c_2)\!=\!\textsf{true},\ t)
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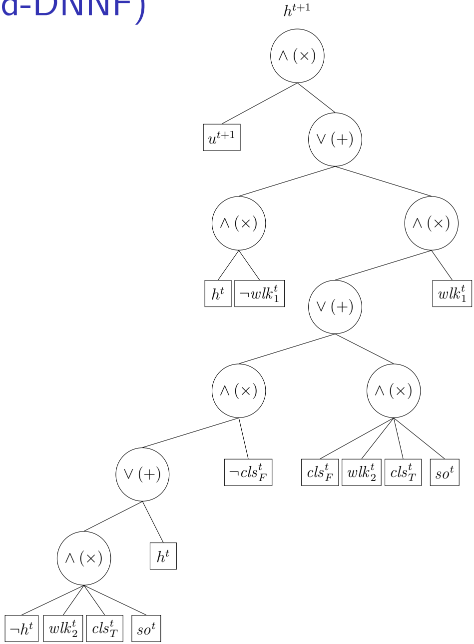

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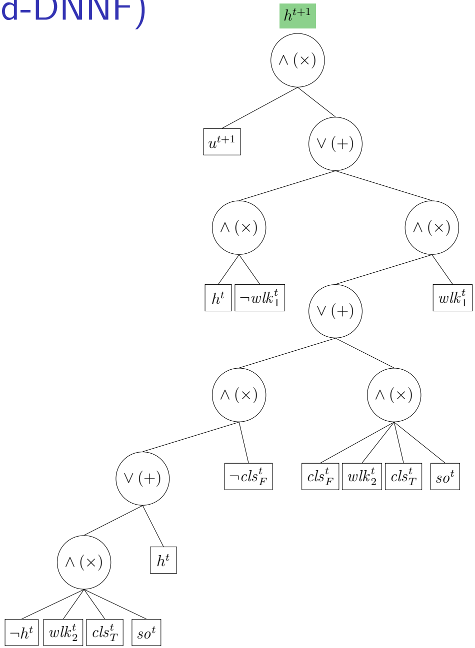

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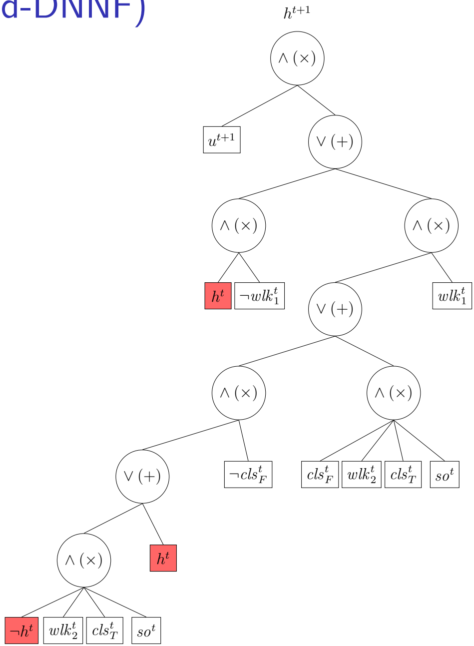

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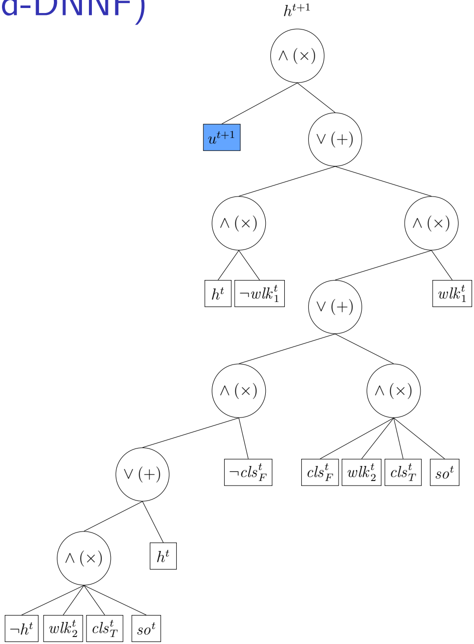

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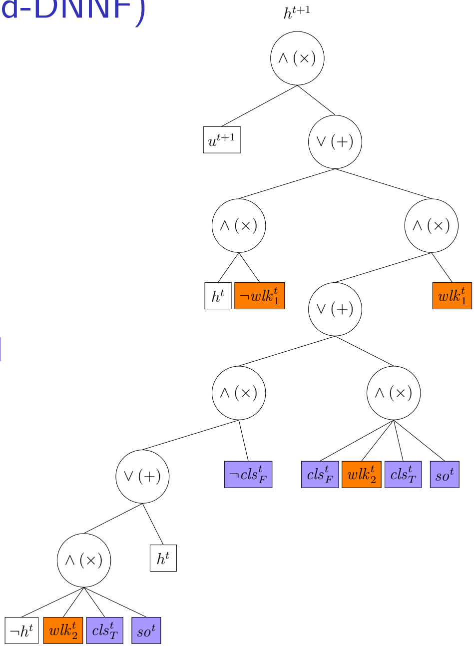

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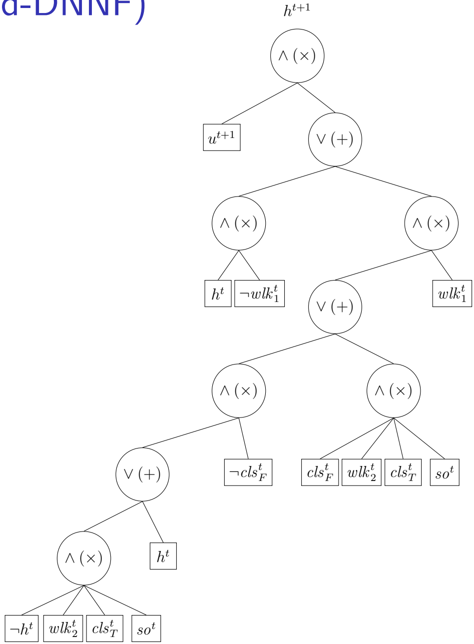
\end{array}
$$

$$
\begin{aligned}
&P(q) = P(h^{t+1}) = \\
&P(u^{t+1}) \times \Bigg[\big(P(h^t) \times P(\neg wlk_1^t)\big) + P(wlk_1^t) \times \\
&\Bigg[\Big[\big[(P(\neg h^t) \times P(wlk_2^t) \times P(cls_T^t) \times P(so^t)) + P(h^t) \big] \times \\
&P(\neg cls_F^t) \Big] + (P(cls_F^t) \times P(wlk_2^t) \times P(cls_T^t) \times P(so^t)) \Bigg] \Bigg] = \ 0.344 \ .
\end{aligned}
$$



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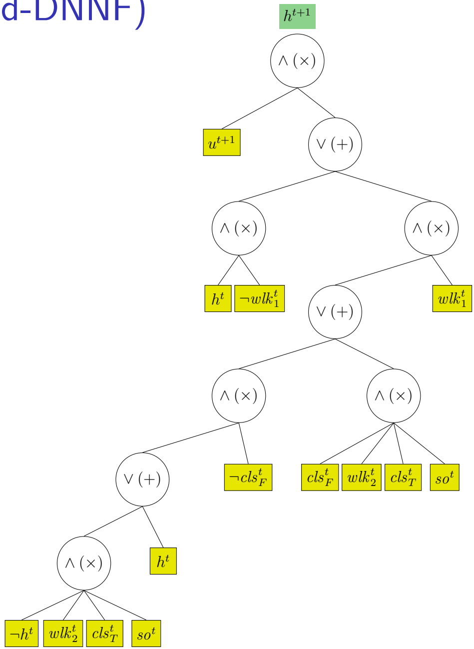
$$P(q) = P(h^{t+1}) =$$

$$P(u^{t+1}) \times \Bigg[\big(P(h^t) \times P(\neg wlk_1^t)\big) + P(wlk_1^t) \times \Bigg[\Big[\big[(P(\neg h^t) \times P(wlk_2^t) \times P(cls_T^t) \times P(so^t)) + P(h^t) \big] \times P(\neg cls_F^t) \Big] + (P(cls_F^t) \times P(wlk_2^t) \times P(cls_T^t) \times P(so^t)) \Bigg] \Bigg] = 0.344 \; .$$



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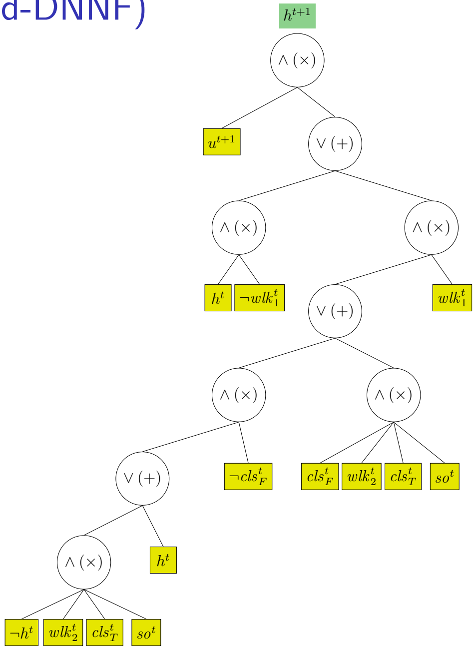

Tensor-pEC: Encoding

- Domain entities: N
- Window time-points: Ω

happensAt$(e(X, Y),\ T)$

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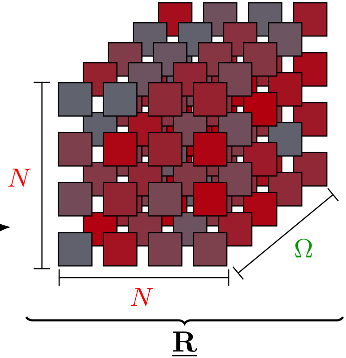
$\underbrace{\text{happensAt}(e(X, Y), T)}_{r}$



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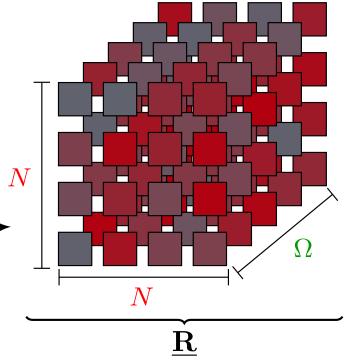
$$\underbrace{\text{happensAt}(e(X, Y), T)}_{r}$$



$$\underline{\mathbf{R}}_{i,j,k} = \begin{cases} p, & \text{if } P(\bigvee_{Proofs(r)}) = p \;\; \text{for } c_i, c_j, t_k \\ 0, & \nexists\, Proof(r) \;\; \text{for } c_i, c_j, t_k \end{cases}$$

$$\forall\, 1 \leq i, j \leq N, 1 \leq k \leq \Omega \;\; .$$

Tensor-pEC: Reasoning

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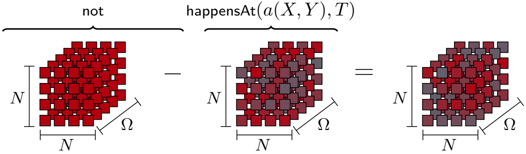
Negation:



Tensor-pEC: Reasoning

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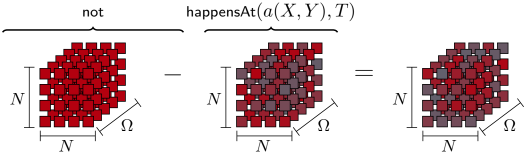
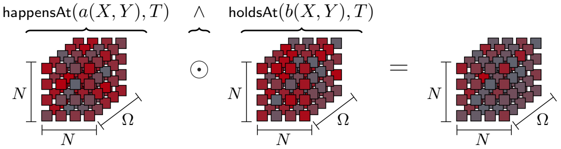


Conjunction:



Tensor-pEC: Reasoning

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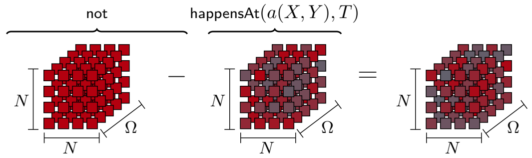

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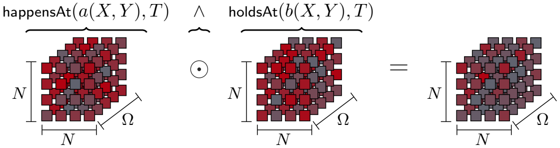
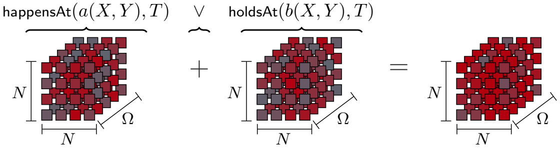
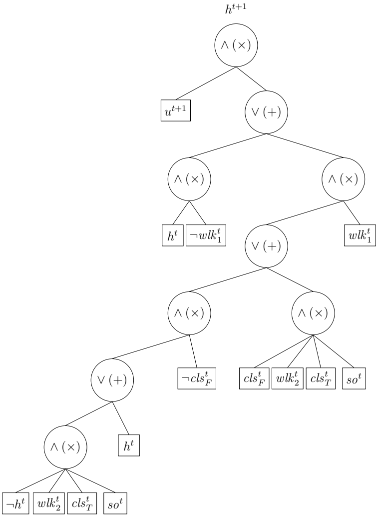


Disjunction:



d-DNNF Factorization



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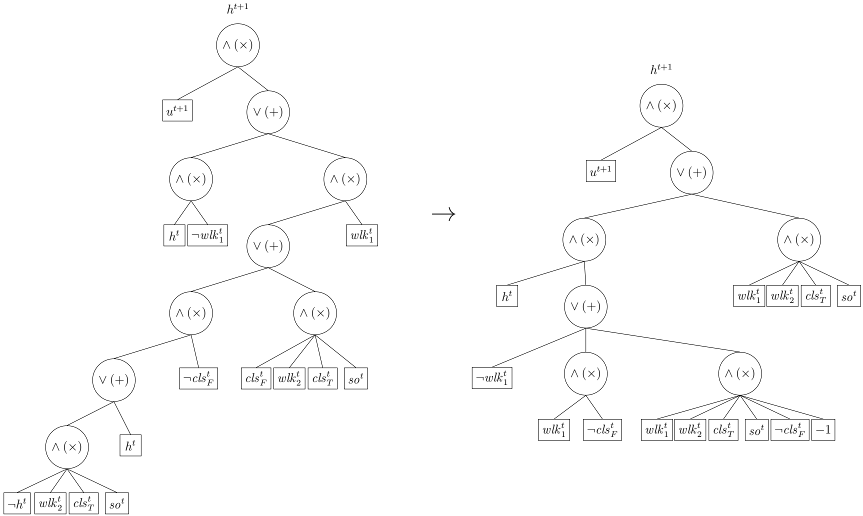

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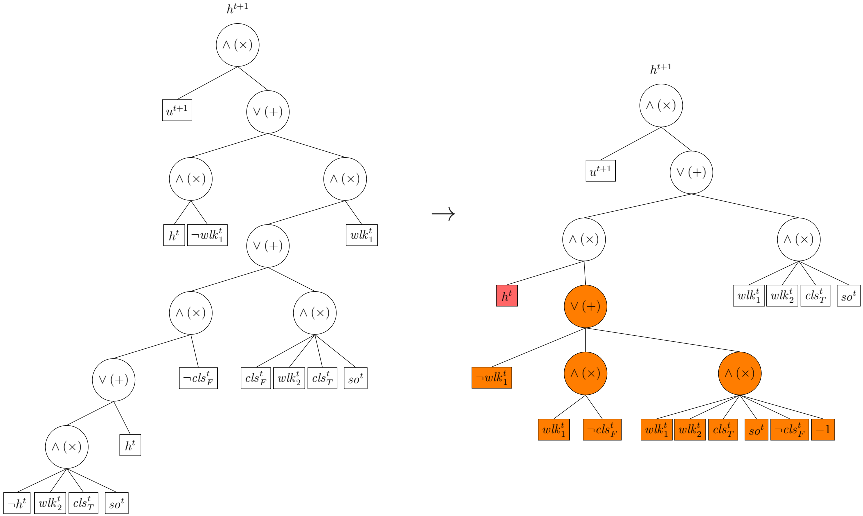

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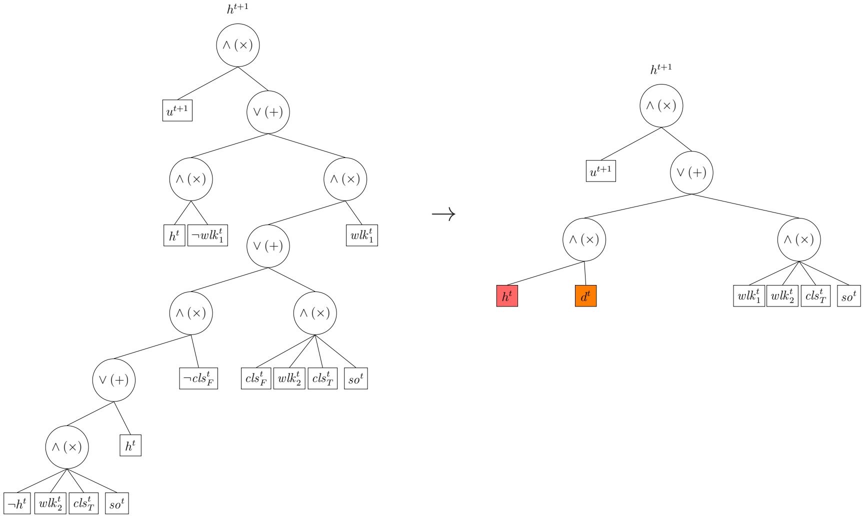

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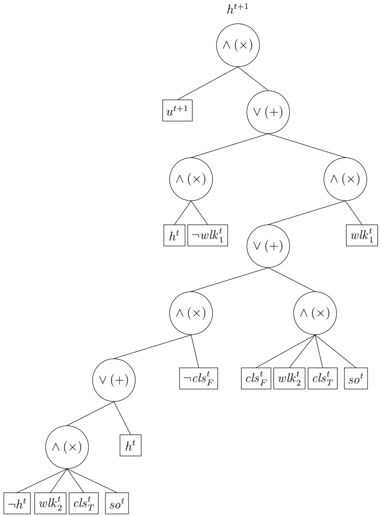

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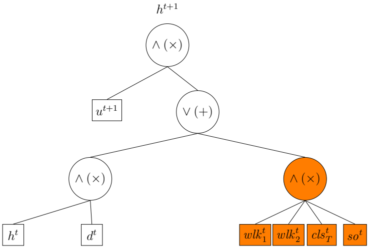

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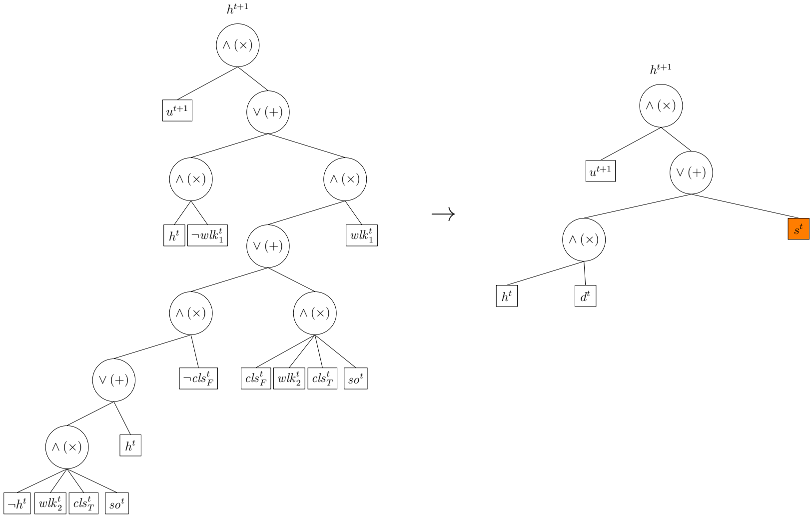
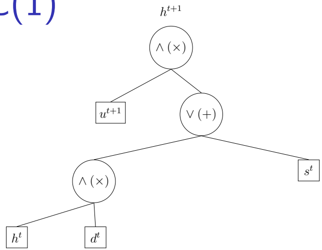


Tensor-pEC: WMC(1)



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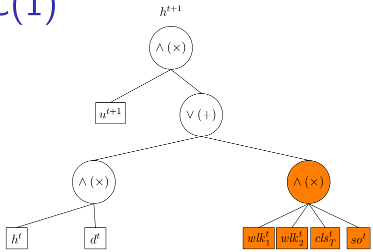

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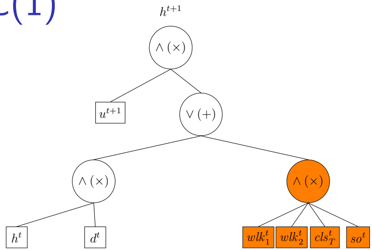

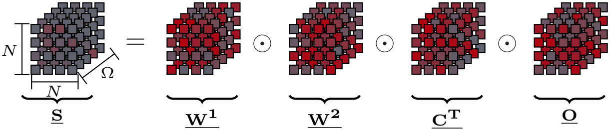

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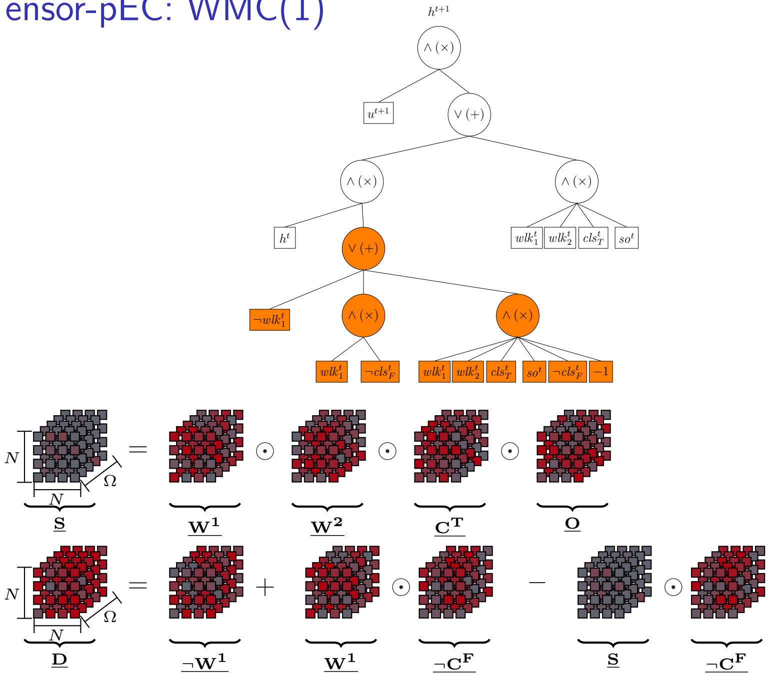
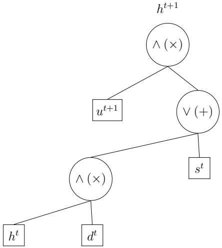


Tensor-pEC: WMC(2)



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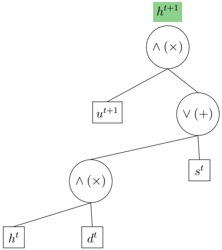

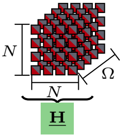

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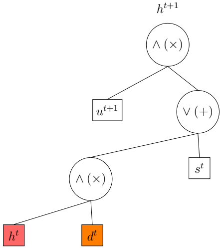
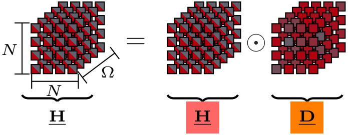
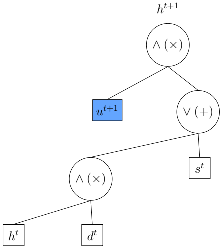
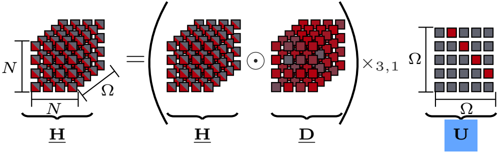


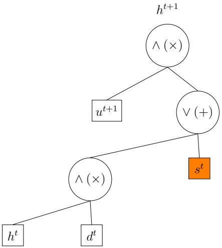


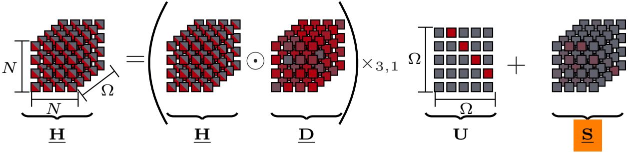
Tensor-pEC: WMC(2)



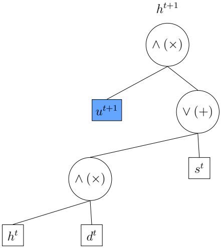


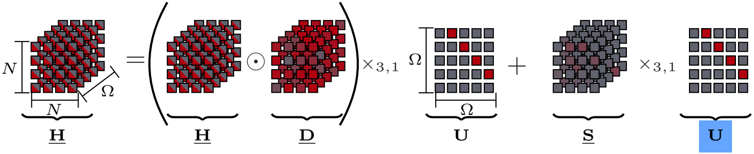
Tensor-pEC: WMC(2)



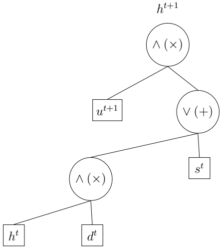


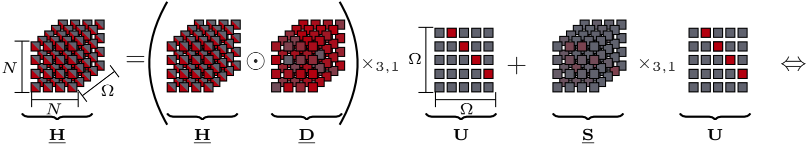
Tensor-pEC: WMC(2)



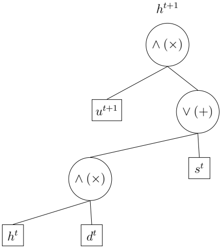


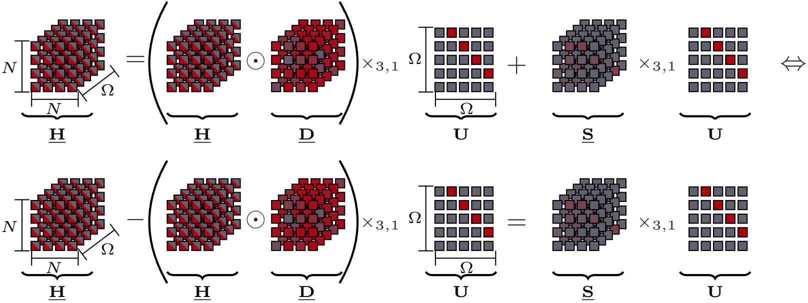
Tensor-pEC: WMC(2)



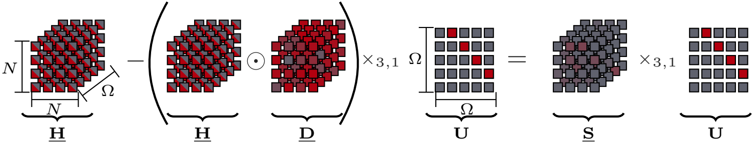


Tensor-pEC: WMC(2)

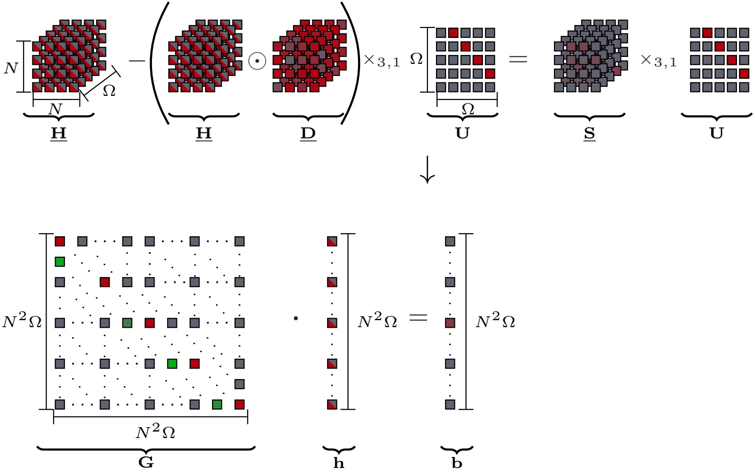




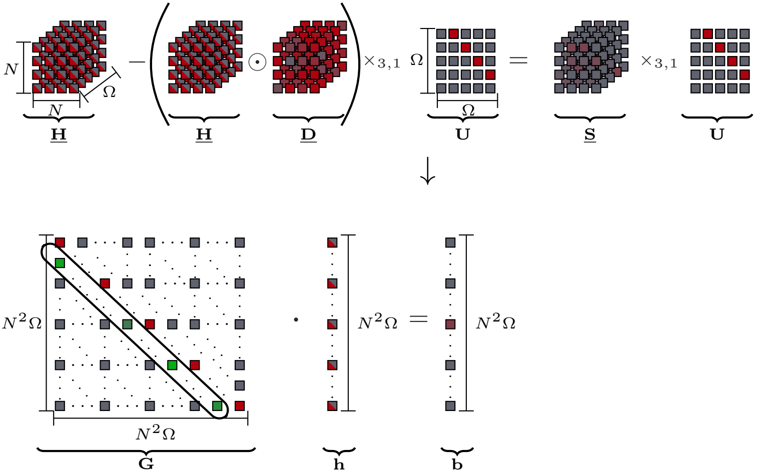
Tensor-pEC: WMC(3)



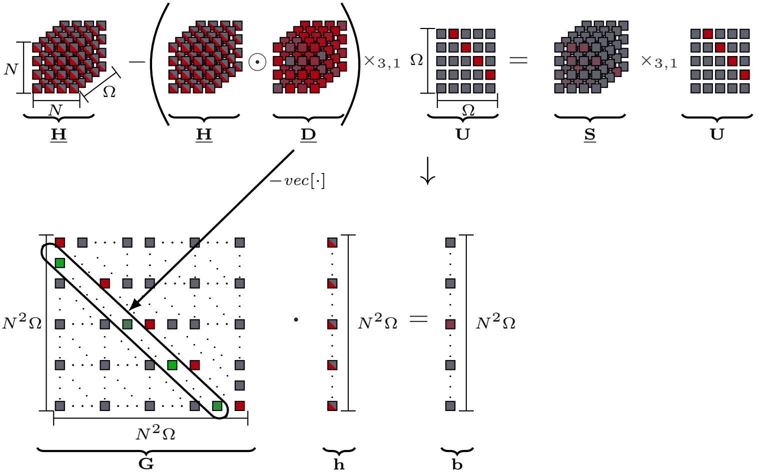
Tensor-pEC: WMC(3)



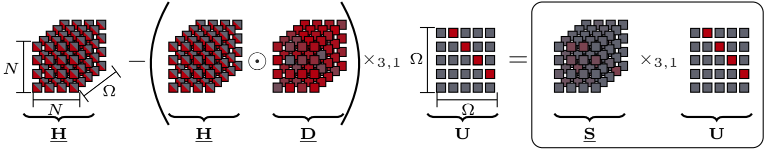
Tensor-pEC: WMC(3)

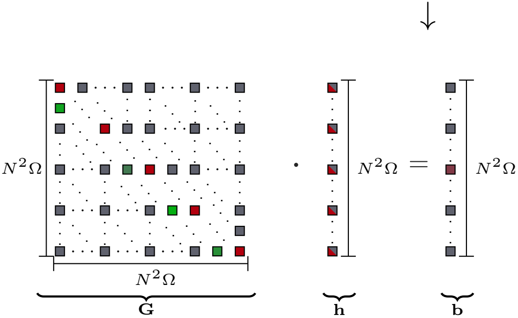


Tensor-pEC: WMC(3)

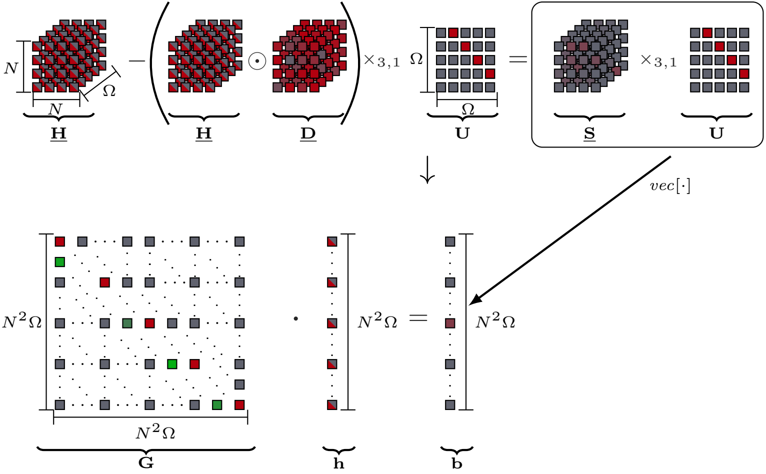


Tensor-pEC: WMC(3)

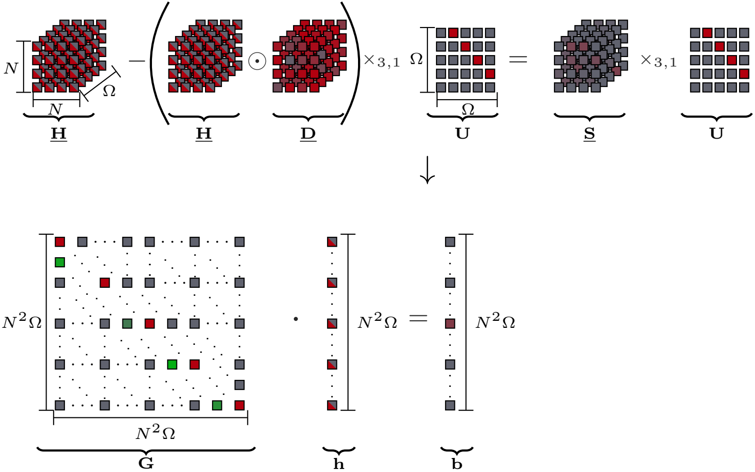




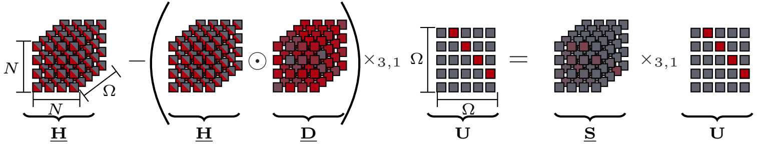
Tensor-pEC: WMC(3)

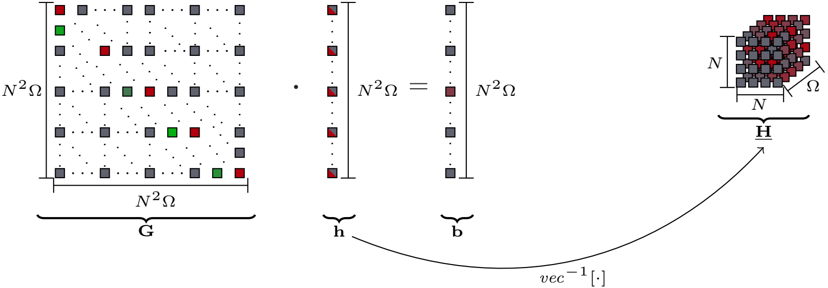


Tensor-pEC: WMC(3)



Tensor-pEC: WMC(3)





Tensor-pEC: Formal Properties

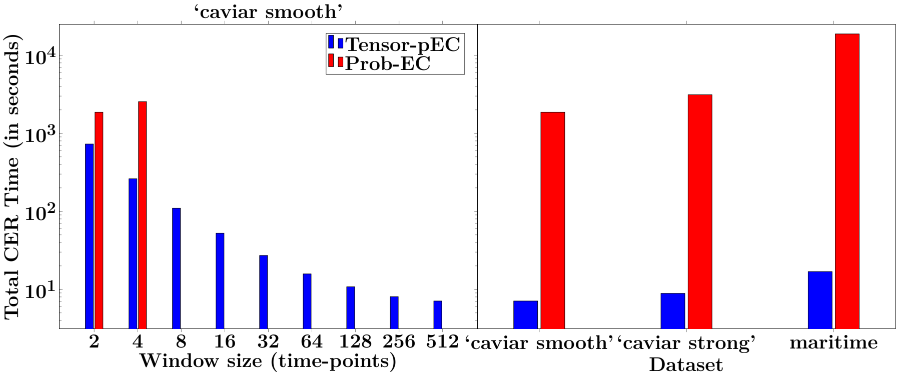
Correctness

The **unique solution** of the equation coincides with the *success probabilities* (WMC) of a fluent-value pair to hold, as computed symbolically.

Empirical Analysis (1)

- Tensor-pEC vs. Prob-EC.
- The experiments were performed on a single core.
- Datasets:
  - CAVIAR:
    - ‘caviar smooth’: noise to input SDEs.
    - ‘caviar strong’: noise both to SDEs and contextual data.
  - Maritime: noise to AIS messages.
- Tensor-pEC: use of sparse representations.
- Code: https://github.com/eftsilio/Tensor-pEC-code

Empirical Analysis (2)



Summary

- Tensor-based Probabilistic Complex Event Recognition:
  - Represents EC probabilistic predicates as tensors.
  - Reasoning through algebraic operations.
  - Computes the success probabilities (WMC) of fluents by solving a linear equation.
  - Formal Properties.
  - Significant performance improvement wrt the symbolic counterpart.

Summary

- Tensor-based Probabilistic Complex Event Recognition:
  - Represents EC probabilistic predicates as tensors.
  - Reasoning through algebraic operations.
  - Computes the success probabilities (WMC) of fluents by solving a linear equation.
  - Formal Properties.
  - Significant performance improvement wrt the symbolic counterpart.
- Next:
  - Exploit parallel algorithms and hardware resources (e.g., GPUs).
  - Scalable neuro-symbolic CER.