NeuroSymbolic AI for Making Sense of Streaming Data: Tensor-based CER

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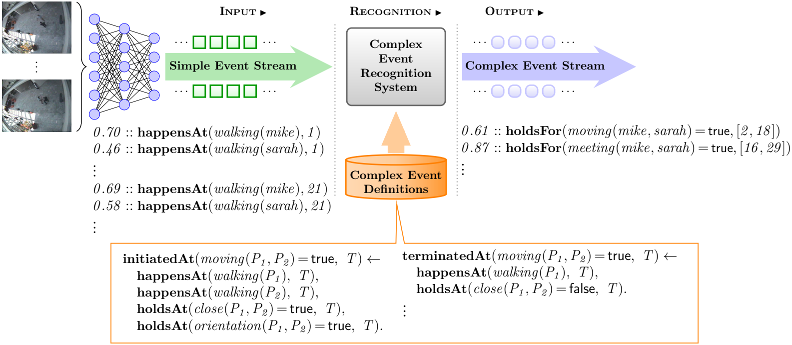
Slides, Papers, Code & Data

StreamingNeSy

Complex Event Recognition lab

Complex Event Recognition (CER)



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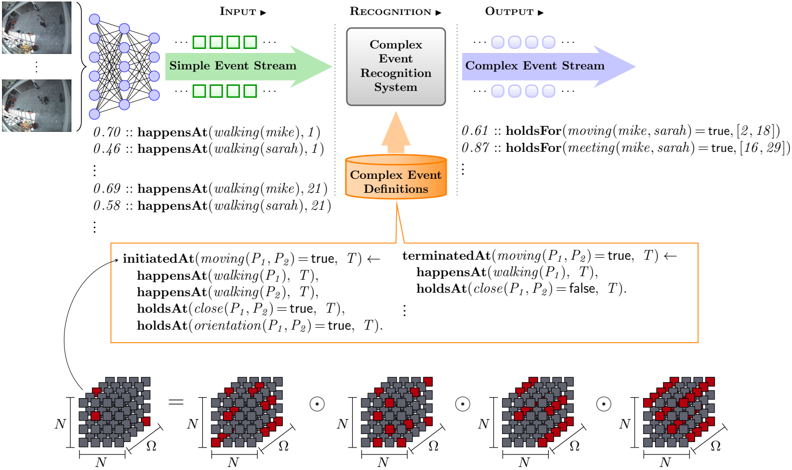

Tensor-based Event Calculus*: Encoding

- Application entities: N
- Window time-points: Ω

happensAt$(e(X, Y),\ T)$

*Tsilionis et al., A Tensor-Based Formalization of the Event Calculus. IJCAI, 2024.

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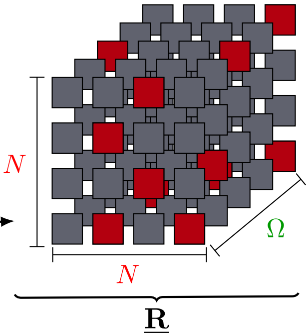

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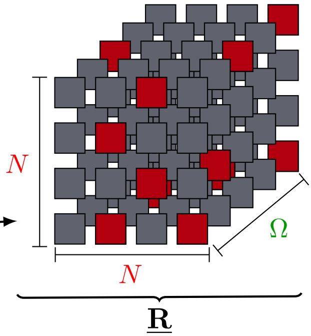

$$\underline{\mathbf{R}}_{i,j,k} = \begin{cases} 1, & \text{if } \mathbf{M}_P \models r, \ \text{for } c_i, c_j, t_k \\ 0, & \text{o.w} \end{cases}$$

$$\forall\, 1 \leq i, j \leq N, 1 \leq k \leq \Omega \ .$$

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Tensor-based Event Calculus*: Encoding (Example)

Assume:

- Two entities c_1, c_2
- Two time-points t_1, t_2
- The following groundings:

$$\begin{aligned}
&\textsf{happensAt}(e(c_1, c_2), t_1)\\
&\textsf{happensAt}(e(c_1, c_2), t_2)\\
&\textsf{happensAt}(e(c_2, c_1), t_2) \quad .
\end{aligned}$$

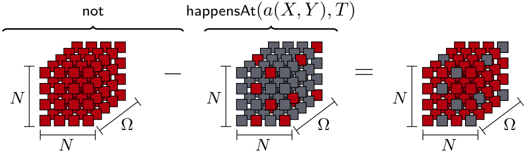
Then the encoding will be:

$$\underbrace{\left[\begin{array}{cc|cc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]}_{\textsf{happensAt}(e(X,Y),T)} \quad .$$

*Tsilionis et al., A Tensor-Based Formalization of the Event Calculus. IJCAI, 2024.

Reasoning: Logical Connectives

Negation:



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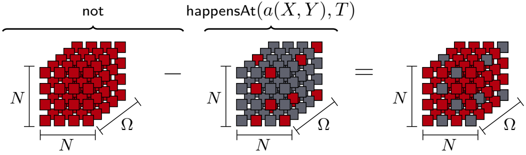
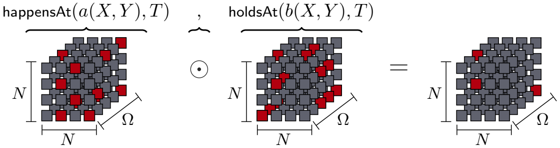


Conjunction:



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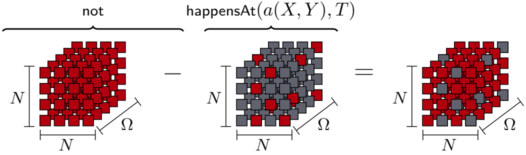

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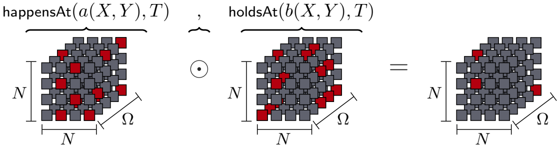
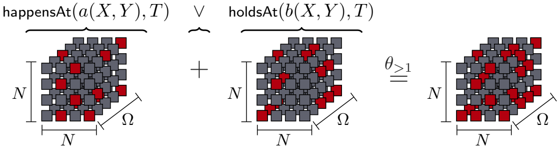


Disjunction:



Reasoning: Rule Evaluation

$$\text{initiatedAt}(\mathit{fl}(X, Y){=}v,\ T)\ \leftarrow$$
$$\quad \text{happensAt}(e(X, Y),\ T)\ \ ,$$
$$\quad \text{holdsAt}(d(X, Y){=}v_d,\ T)\ \ .$$

Reasoning: Rule Evaluation

$\mathsf{initiatedAt}(\mathit{fl}(X, Y){=}v,\ T) \leftarrow$

$\mathsf{happensAt}(e(X, Y),\ T)$,

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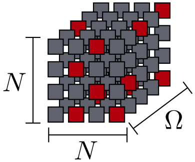

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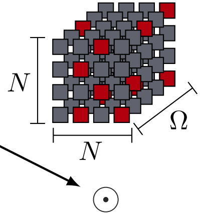

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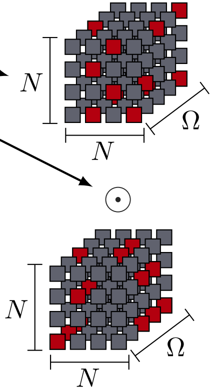

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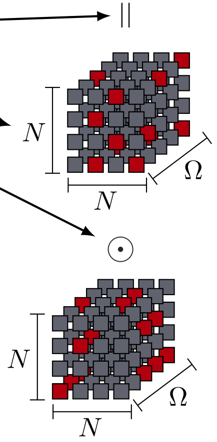

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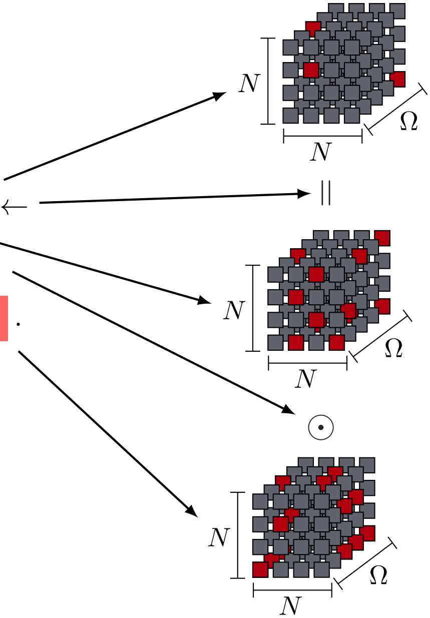

Reasoning: Rule Evaluation (Example)

$$\underbrace{\left[\begin{array}{cc|cc} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{array}\right]}_{\textsf{happensAt}(e(X,Y),\, T)} \odot \underbrace{\left[\begin{array}{cc|cc} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]}_{\textsf{holdsAt}(d(X,Y)=v_d,\, T)} = \underbrace{\left[\begin{array}{cc|cc} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]}_{\textsf{initiatedAt}(\mathit{fl}(X,Y)=v,\, T)}$$

Model Computation

$\mathsf{holdsAt}(\mathit{fl}(X,Y){=}v,\ T) \leftarrow$
$\quad \mathsf{initiatedAt}(\mathit{fl}(X,Y){=}v,\ T_{prev}),$
$\quad \mathsf{not}\ \ \mathsf{terminatedAt}(\mathit{fl}(X,Y){=}v,\ T_{prev}),$
$\quad \mathit{next}(T_{prev}, T).$

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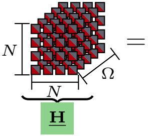

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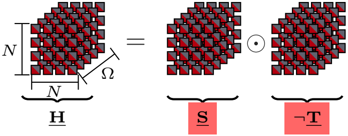

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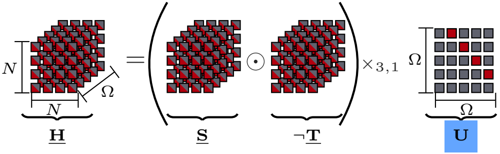

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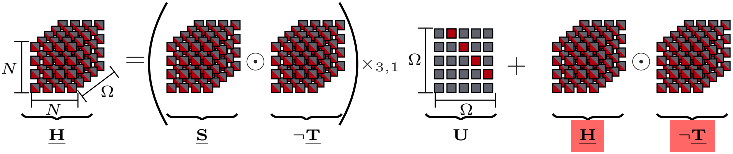

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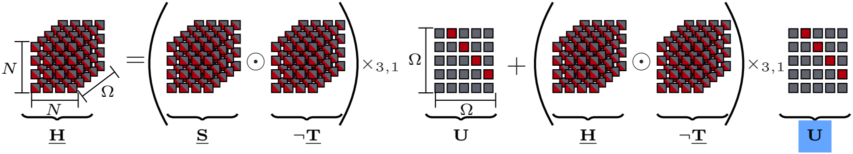

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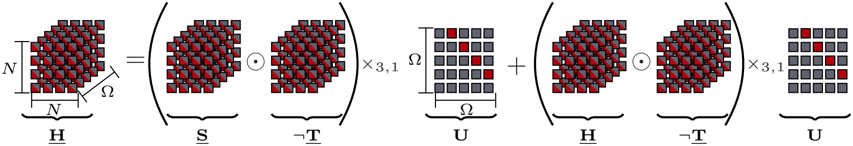


Equivalence with Symbolic Reasoning

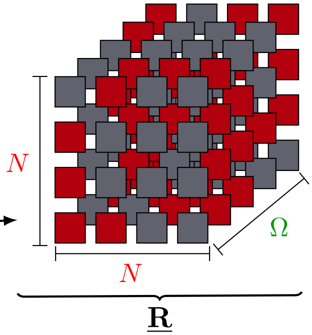
Correctness

The tensor-based Event Calculus computes the same time-points for each FVP as RTEC.

Tensor-based Probabilistic Event Calculus

- Application entities: N
- Time-points: Ω

$$\underbrace{\text{happensAt}(e(X, Y),\ T)}_{r}$$



Crisp[†]:

$$\underline{\mathbf{R}}_{\,i,j,k} = \begin{cases} 1, \text{ if } \mathbf{M}_P \models r, \ \text{ for } c_i, c_j, t_k \\ \\ 0, \text{ o.w} \end{cases}$$

$$\forall\ 1 \leq i, j \leq N, 1 \leq k \leq \Omega\ .$$

[*]Tsilionis et al., A Tensor-Based Formalization of the Event Calculus. IJCAI, 2024.

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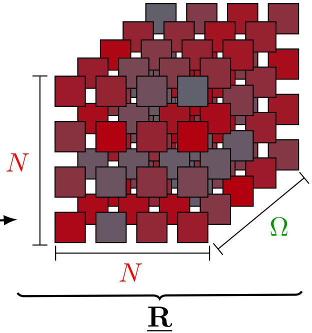

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$$\forall\, 1 \leq i, j \leq N, 1 \leq k \leq \Omega\ .$$

Probabilistic[‡]:

$$\underline{\mathbf{R}}_{\,i,j,k} = \begin{cases} p,\ \text{if } P(\bigvee_{Proofs(r)}) = p\ \ \text{for } c_i, c_j, t_k \\ \\ 0,\ \nexists\ Proof(r)\ \ \text{for } c_i, c_j, t_k \end{cases}$$
$$\forall\, 1 \leq i, j \leq N, 1 \leq k \leq \Omega\ .$$

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[*] Tsilionis et al., A Tensor-Based Formalization of the Event Calculus. IJCAI, 2024.

[†] Tsilionis et al., A Tensor-Based Probabilistic Event Calculus. KR, 2025.

Summary

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- Reason through algebraic operations.

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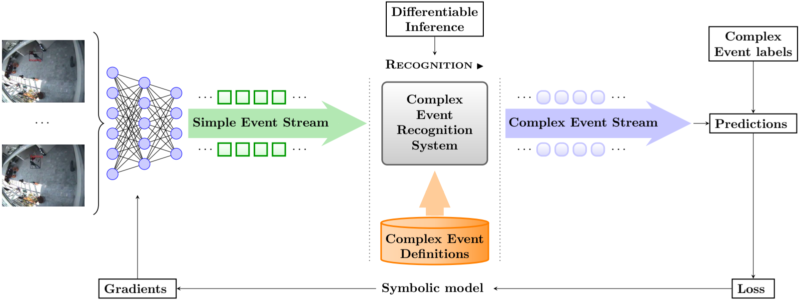
Tensor-based CER:

- Transform symbolic representation into tensors.
- Reason through algebraic operations.
- Maintain formal semantics.
- Maintain treatment of uncertainty.
- Performance improvement wrt the symbolic counterpart.

Next: Scalable Neuro-Symbolic CER



Course Structure

- Introduction to Complex Event Recognition (CER).
- Formal Models for CER
- Probabilistic CER.
- Tensor-based CER.

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- Introduction to Complex Event Recognition (CER).
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- Topics not covered.